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Scaling law in target-hunting processes

Shi-Jie Yang
Department of Physics, Beijing Normal University, Beijing 100875, China

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We study a hunting process for a target, in which the hunter tracks the goal by smelling odors it emits. The odor intensity is supposed to decrease with the diffusion distance. The Monte Carlo experiment is carried out on a two-dimensional square lattice. Having no idea of the location of the target, the hunter determines its moves only by random attempts in each direction. By sorting the searching time in each simulation and introducing a variable x to reflect the sequence of searching times, we obtain a curve with a wide plateau, indicating the most probable time of successfully finding the target. The simulations reveal a scaling law for the searching time versus the distance to the position of the target. The scaling exponent depends on the sensitivity of the hunter. Our model may be a prototype in studying such searching processes as various food-foraging behaviors of wild animals.

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In the past years, the diffusion-controlled reactions have been extensively studied through random-walk models. Such applications range from chemical processes, electronic scavenging and recombination, to electronic and vibrational energy transfer in condensed media [1–9]. Many works have been devoted to the target annihilation problem, in which randomly placed targets are annihilated by random walkers, and its dual, the trapping problem [10,11]. Other models treat hindered diffusion problems that involve random point obstacles [12]. In these models, the tracer moves from site to site on a lattice and falls into wells of various depths at the sites. The tracer does not know the depth of a well before it enters. Another possibility is a mountain model, in which all sites are at zero energy and the barriers are on the bonds joining the sites [13,14]. Generally, random-walk models are ideally suited for computer simulations, a practical way to obtain results, since for the vast majority of cases no purely analytical method exists.

In this work, we focus on another class of random-walk problems. We study the so-called target-hunting processes, which frequently occur in biological systems, such as a shark searching for food by smelling the blood in the ocean, or honeybees flying in the countryside to locate the foraging nectars [15–17], or in metabolic processes such as cell motion and chemotaxis [18–22]. It can be viewed as a target-oriented problem, in which the hunters try to reach the targets by following some kind of behavior rules. In our model, an active hunter is trying to find out a target that emits a special kind of odor. The Monte Carlo simulations are carried out on a two-dimensional square lattice. Since neither the distance nor the direction of the target is presumed to be known, the searcher should determine its moves by random attempts in each direction, just as a snake turns its head from side to side to test the variation of odor intensity. There is some chance for the hunter to move in the wrong direction because of randomness. Hence it is not a traditional biased random walk. After sorting each searching process in a time sequence, we obtain a curve with a wide plateau, indicating the most probable time of successfully finding the target. By fitting the numerical results, we find a scaling law for the searching time dependence on the distance to the position of the target. The scaling exponent is found to be dependent on the sensitivity of the hunter. We consider that this scaling law rarely happens in ordinary biased random walks.

The game rules are as follows. The hunter at the origin O is trying to find a target that emits a special kind of odor. Since the hunter has no way to know the location of the target, it randomly moves around its original position to test the variation of the odor intensity. z_0 is the present distance of the hunter to the target while z_1 is the corresponding distance of the next attempted step. The Monte Carlo steps are implemented as follows. If $(z_0/z_1)^{\alpha}>\zeta$, where the parameter α reflects the sensitivity of the hunter and ζ is a random number, then the attempt is accepted. Otherwise it is refused. This rule implies that the intensity of the signal emitted by the target is inversely proportional to the distance of the hunter to the target. Other choices of the relation do not alter the result qualitatively. In this way, the hunter approaches the goal in a stochastic style.

Figure 1 displays a typical route of the hunter searching for the target on a regular lattice. When the hunter is far away from the goal, the ratio z_0/z_1 is close to 1. Most of the moving attempts are accepted, even the hunter walks in the wrong direction. The hunter appears to linger around for quite a while. Hence the motion of the hunter is nearly a

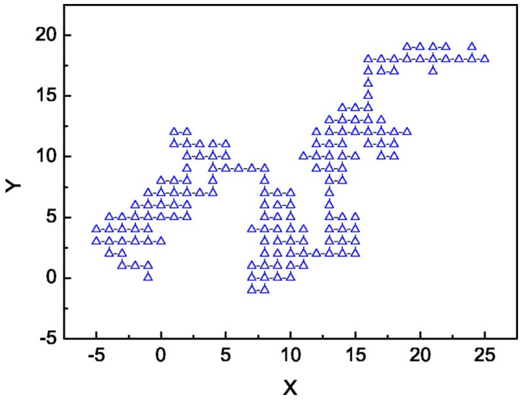

FIG. 1. A typical hunting route on a square lattice. The start point is at the origin and the target is at (25, 19).

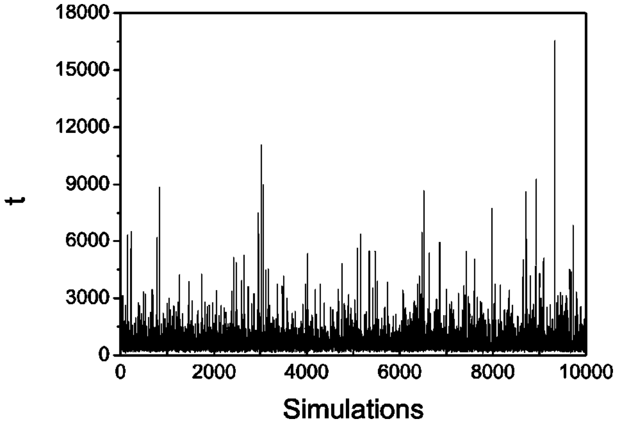

FIG. 2. Searching time for 10 000 simulations. The original distance to the target is $z=31.4$ and the sensitivity parameter $\alpha=6$.

Brownian random walk. As the goal becomes nearer, the ratio of z_0/z_1 gradually approaches 0.5 and the probability of a move in the wrong direction being refused increases. Hence the searching route seems more straightforward.

Figure 2 shows the searching time for each simulation for a distance of $z=31.4$ and $\alpha=6$. It is understandable that the searching times are different for different stochastic processes. The distribution is not like a white noise. There are large fluctuations away from the most probable searching time. In Fig. 3, we plot the distribution of searching times for $z=31.4$, 65.6, 137.1, 188.4, respectively. It is seen that the distribution is not of the Poissonian form. The curve has a very long time tail. Instead, a power relation is found for the maximal value of the distribution with the most probable time,

$$V_m \sim t_p^{-d_m}, \tag{1}$$

with $d_m=1.05$ for $\alpha=6$.

There is another power relation between the distance of the target and the most probable searching time,

$$t_p \sim z^{d_p}, \tag{2}$$

where $d_p=1.77$ for $\alpha=6$.

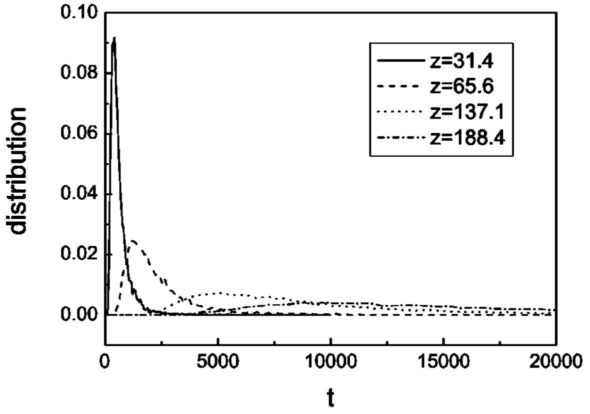

FIG. 3. Searching time distribution for distances $z=31.4, 65.6, 137.1, 188.4$, respectively. $\alpha=6$.

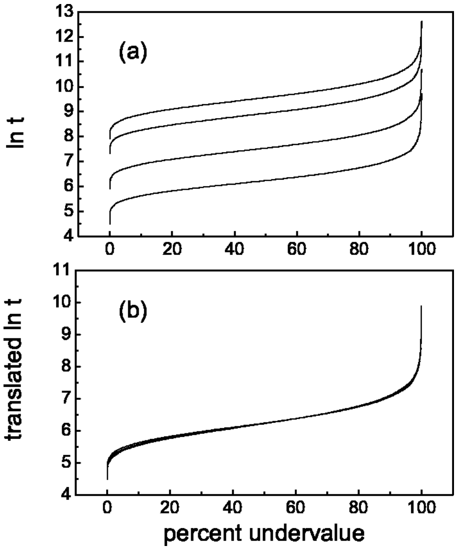

FIG. 4. Time-sorted curves for 10 000 simulations. The horizontal axis x is the sequence of searching times represented as a percentage and the vertical axis is logarithmic time. $\alpha=6$. (a) is for various distances z from the origin, with $z=31.4, 65.6, 137.1, 188.4$ from bottom to top. Evidently, the curves consist of three parts, and a wide linear region is formed. These curves are parallel to each other. (b) shows that after proper displacement all the curves collapse into one.

In Fig. 4 we redistribute the data in Fig. 2 by sorting with increasing time. Figure 4(a) is for various distances z from the origin, with $z=31.4$, 65.6, 137.1, 188.4 from bottom to top and $\alpha=6$. The horizonal axis is the sequence of searching times represented as a percentage. A wide plateau is formed in the intermediate range. Figure 4(b) shows that after proper displacement all of the curves collapse into one, implying that these curves are parallel to each other. Hence each curve can be described by a single function $f(x,\alpha)$ plus a z-dependent function $\phi(z,\alpha)$,

$$\ln t(x,z,\alpha) = f(x,\alpha) + \phi(z,\alpha) \tag{3}$$

with x the sequence of searching times represented as a percentage. The function $f(x)$ is Arabic ogivelike. It can be checked that

$$\ln t(\sqrt{z_1 z_2},x) = \frac{1}{2}[\ln t(z_1,x) + \ln t(z_2,x)]. \tag{4}$$

Formula (3) can be written as

$$\ln t(x,z,\alpha) = f(x,\alpha) + \eta(\alpha)\ln z. \tag{5}$$

In Fig. 5 we study the dependence of the searching time with respect to the sensitivity parameter α. Figure 5(a) shows the curves for $\alpha=4,8,12,16,20$. After properly rescaling the curves in (a) by times $\ln t$ with a coefficient α^{β}, where β is determined below, all of the curves become parallel. From Fig. 5(b), we deduce $\alpha^{\beta}\ln t=\tilde{f}(x)+\tilde{\phi}(z,\alpha)$. By comparing with Eq. (5), one gets

$$\ln t(x,z,\alpha) = \alpha^{-\beta} f(x) + \eta(\alpha)\ln z. \tag{6}$$

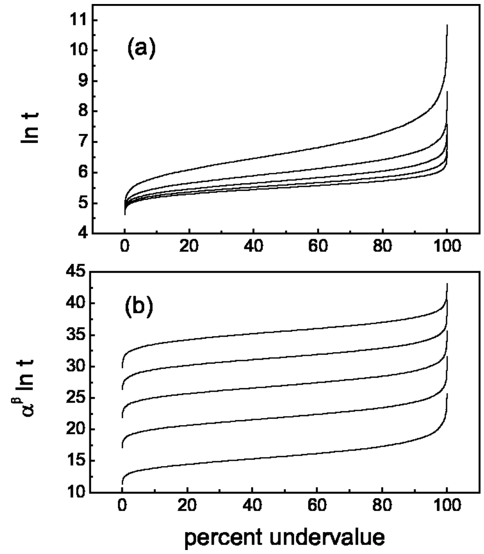

FIG. 5. Time-sorted curves for 10 000 simulations for various parameter values α. The horizontal axis x is the sequence of searching times represented as a percentage and the vertical axis is logarithmic time. The original distance is fixed at $z=31.4$. (a) is for $\alpha=4,8,12,16,20$ from top to bottom. (b) shows the rescaled curves of (a) for $\alpha=4,8,12,16,20$ from bottom to top. These curves are parallel to each other.

The index β can be derived by considering the dependence of the slope k_2 of the plateau in Fig. 5(a) on parameter α. There is a good linear relation between $\ln k_2$ and $\ln \alpha$, as show in the inset of Fig. 6,

$$\ln k_2 \sim -\beta \ln \alpha. \tag{7}$$

We measured $\beta=0.623$. It is noteworthy that β is a constant independent of the sensitivity parameter α. It results from the stochastic process.

Finally, we tried to find the relation between η and α. Figure 7(a) depicts the relation of $\ln t$ versus $\ln z$ at $x=0.6$ for various parameters α. From Fig. 7(b),

$$\ln \eta(\alpha) \sim -\delta\alpha \tag{8}$$

with $\delta=0.01$.

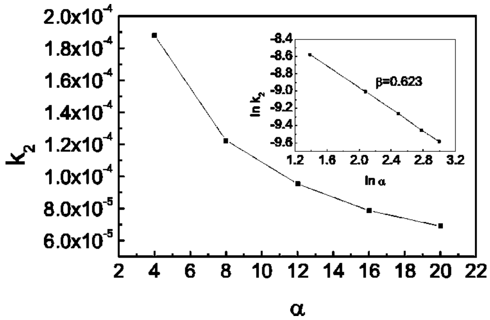

FIG. 6. The dependence of the slope of the plateau in Fig. 4 on the parameter α. The distance $z=31.4$. Inset: the linear relation of the logarithmic slope k_2 with logarithmic α.

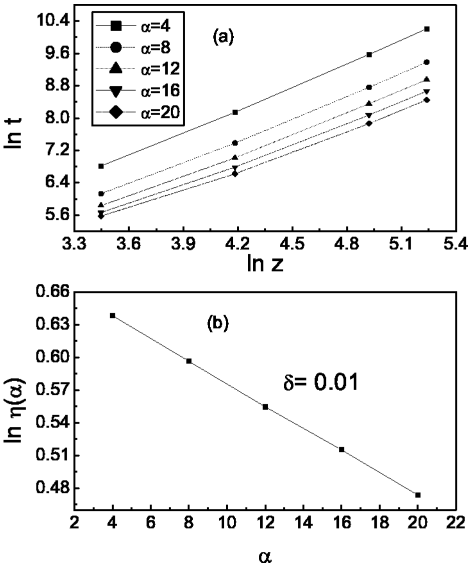

FIG. 7. (a) A plot of $\ln t$ versus $\ln z$ at $x=0.6$ for various parameters α. (b) The linear relation of $\ln \eta(\alpha)$ with respect to α.

Combining all the above factors, we consequently obtain a complete relation for the searching time with respect to the distance as well as the sensitivity parameter α,

$$\ln t(x,z,\alpha) = \alpha^{-\beta} f(x) + c_0 e^{-\delta\alpha} \ln z, \tag{9}$$

or

$$t(x,z,\alpha) = e^{f(x)/\alpha^{\beta}} z^{d}. \tag{10}$$

We find that there is a generalized scaling law between t and z with exponent

$$d = c_0 e^{-\delta\alpha}. \tag{11}$$

From formula (2), $\alpha=6$, $d=d_p=1.77$. We get $c_0=1.88$. This shows that the power-law exponent is α dependent. As α increases from zero to infinity, the exponent decreases from 1.88 to zero. In Eq. (10), the contributions of variable x, which sorts the searching time in each simulation, are completely merged into a prefactor and the scaling exponent is x independent. It should be noted that there is a functional relation between the searching time and the distance in such stochastic processes.

In summary, we introduced a variable x to denote the sequence of searching times. We plot a curve with a wide plateau, indicating the most probable time of successfully finding the goal. Instead of calculating the mean square root, we introduce a sorting parameter x to figure out an analytical expression. The simulations reveal a scaling law for the searching time versus the distance to the position of the target. The scaling exponent is dependent on the sensitivity of the hunter. We believe that our treatment of the statistical data may be useful in other cases. The existence of the scaling law may have implications for the possibility for the hunter to walk in a wrong direction or stay at the same place for quite a while. This rarely happens in an ordinary biased random walk. We point out that the results are valid not only on the square lattice, but also for continuous movement (with fixed step length) in the two-dimensional plane. However, the explicit form of the function $f(x)$ is still lacking. It is also desirable to deduce an analytical expression for Eq. (10) from the first principles of statistics.

We suggest that the scaling law in the hunting process may be an additional behavior rule in the food-foraging processes of wild animals, which has not attracted much attention. In turn, verification of the law from direct observations by zoologists or entomologists is also expected. Our target-oriented model may be a prototype in studying the food-foraging processes in wildlife as well as in other searching games.

ACKNOWLEDGMENT

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